



# Logic

## Course Syllabus

### Course Requisites

|   |   |
|---|---|
| <b>Educational degree</b>                   | <i>First (bachelor's degree)</i>  |
| <b>Knowledge domain</b>                     | <i>General</i>  |
| <b>Program</b>                              | <i>General</i>  |
| <b>Educational program</b>                  | <i>General</i>  |
| <b>Course status</b>                        | <i>Optional</i>   |
| <b>Form of Education</b>                    | <i>Full-time/External/Part-Time</i>   |
| <b>Academic year, semester</b>              | <i>2<sup>nd</sup> academic year, 1<sup>th</sup> and 2<sup>nd</sup> semesters</i>  |
| <b>Discipline volume</b>                    | <i>2 credits ECTS (60 hours) / Lectures (18 hrs) / Seminars (18 hrs) / Individual work (24 hrs)</i>   |
| <b>Final assessment/ evaluation methods</b> | <i>Knowledge Survey</i>   |
| <b>Schedule of the Classes</b>              |   |
| <b>Learning Language</b>                    | <i>English</i>  |
| <b>About the Instructor(s)</b>              | Lectures: <i>candidate of philosophical sciences, lecturer, Mstislav Kazakov (mstkazakov@gmail.com; +380930562957)</i><br>Practical classes: <i>candidate of philosophical sciences, lecturer, Mstislav Kazakov (mstkazakov@gmail.com; +380930562957)</i> |
| <b>Course URL</b>                           | <a href="https://classroom.google.com/c/MjU0MjYxODMwODgz">https://classroom.google.com/c/MjU0MjYxODMwODgz</a>   |

### Educational Program

#### 1. Course description, Aims and Scope, Learning outcomes

The *main objective* of the course 'Logic' is an introduction to contemporary areas of research which in manifold extend intersect with laws, algorithms, patterns of human thought, inferencing and reasoning as an attempt to specify the demarcation line between 'sentience' and 'sapience'. According to such a tasks, the main *subject* of the discipline deals with: automated reasoning; natural deduction; Artificial Intelligence (AI) research and development; theory and practice of proofs; formal modelling theory and practice; metamathematics; theories of truth; patterns of human thought (up to some meta-theoretical aspects concerning logic programming and introduction to knowledge bases, expert systems and knowledge representation methods).

The following *program results* are obtained after the familiarization with the course:

- formal representations of the algorithms of human reasoning and inferencing;
- determining the validity of arguments and checking the input data for errors and invalidity;
- performing inductive reasoning procedures in scientific research and professional activity;
- learning theory and using several techniques of formal and informal proofs within the formal systems, such as:
  - 'Gentzen-style' Sequent Calculus;
  - 'Fitch-style' proofs;
  - Tableaux methodology;
  - Automatic resolution systems etc.;
  - improving operating skills concerning the concepts of models for the formal systems;
  - calculating the truth-contingence of propositional functions, simple and compound propositions;
  - building truth tables for any Boolean formula;
  - building Venn's diagrams for the argument validity evaluation;

- building and using logical matrices for many-valued logics;
- operating with the elements of First- and Second-Order Predicate Logics and Modal logics;
- apply rules of inference for theoretical (reasoning) and practical tasks;
- decipher symbolic expressions / modeling semantics for the interpretation of Propositional Functions and Compound Propositions;

## **2. Prerequisites and Corerequisites of a Course**

*The course is optional and, while being oriented on polytechnic profiles, is accessible to any bachelor degree program enrollees. That is, no special knowledge is required. Among the desirable (but not obligatory) skills and knowledge there may be mentioned basic understanding of Set Theory, pre-calculus, mathematical proofs basics, automata theory basics and insights into semantics. The recommended level of English for the successful comprehension is B1.*

The course is organized and structured in soundness with the paradigm of polytechnical education, having in mind the inclusion of truly useful parts of vast area of logical investigations for this paradigm. The knowledge and practical assignments are intended to serve as one of the access points (of introduction) to such areas of further professional education as Data Science, Fuzzy sets and Automata theory, Expert systems, AI Research, Machinic reasoning, Machine Learning, cognitive sciences areas, Neuron Networks and Consciousness applied studies withing Mind-Modelling Paradigm. This is only the list of paradigmatic disciplines, not mentioning the secondary, to which the course 'Logic' may serve as introduction because of wide area of applications of formal logic in contemporary scientific and technological investigations, which is constantly increasing.

## **3. Course Summary**

### **PART I. CLASSICAL PROPOSITIONAL LOGIC**

#### **A. PATTERNS OF REASONING**

1. *Knowledge, Belief, Opinion. Demarcation Problem.*
2. *Patterns of Reasoning. Arguments and Propositions. Logic as a Study of Thought.*
3. *'Laws of Thought': the Rise of Propositional Logic. Rules of Transformation.*

#### **B. FORMAL LOGIC: THE ESSENTIALS**

4. *Introduction to Classical Propositional Logic. Syntax, Semantics, Valuations. Truth Tables.*
5. *Consistency, Soundness, Completeness, Validity. Logical Entailment.*

#### **C. INSIGHTS INTO PROOF THEORY**

6. *Proof Systems. Logic as a Proof Theory. Formal System: the Intro. Rules of Inference.*

### **PART II. FIRST ORDER PREDICATE LOGIC**

#### **A. PROOF THEORY: ADVANCED TOPICS**

7. *Axiomatic Systems. Logic as Metamathematics. Logicism and Formalism. Intuitionism and Constructivism. Rules of Inference, Axioms, Axiom Schemata, Theorems in FS.*
8. *Natural Deduction Systems of Proof: Two-Column Proofs (Fitch, Lemmon, Suppes); Sequent Calculus;*

#### **B. BEYOND THE PROPOSITIONAL CALCULI**

9. *First Order Predicate Logic. Constants, Variables, Quantifiers, Unary Predicates (the Fragment 'Monadic PL').*
10. *Relations and Functions: n-ary predicates. Domain as a key notion of FOPL.*
11. *Model Theory: Interpretations, Valuations and Semantics with Quantifiers.*
12. *FOPL: Inference Rules and Equivalences.*
13. *Quantification with Identity. Rules and Proofs. Identity Theory ABCs.*

#### **C: DECIDABILITY PROBLEM**

14. *Proof Theory Expanded: Tree Tableaux Method.*
15. *Decidability Problem. 'FOPL is semi-decidable'. CPL decidability.*
16. *Resolution Procedure*
17. *Godel's Proofs of Incompleteness.*

## **PART III. NON-CLASSICAL LOGICS**

### **A. MODAL LOGIC AND POSSIBLE WORLDS**

18. *Normal Modal Logics: Classical, Epistemic, Linear Temporal PL,*
19. *Deontic, Temporal and Alethic Logics.*
20. *Quantification of Modal Logic.*
21. *Kripke Semantics of Possible worlds.*

### **B. MANY-VALUEDNESS**

22. *3-valued logics: the Idea. (L3, B3, Kw3, Ks3, Pi3, G3)*
23. *FDE Logics Family (Belnap, Steele, Priest, G4)*
24. *n-valuedness. Alternatives to more-than-2-valuedness.*
25. *Fuzzy Sets and Fuzzy Logic.*

### **C. APPLIED LOGIC AND BEYOND**

26. *Representing the Knowledge: Fact, Data, Knowledge Bases.*
27. *Use of FOPL in Expert Systems and Semantic Frames etc.*
28. *Automated Reasoning. Resolution Method. Provers of Theorems. SAT.*
29. *Formal systems in Technology, Programming, Applied sciences and beyond.*

## **4. Course Texts and/or Other Study Materials**

### **4.1. Course Textbooks:**

1. *Barker-Plummer D., Barwise J., Etchemendy J. Language, Proof and Logic. 2<sup>nd</sup> edition. CSLI Publications, 2011.*
2. *Bergmann M., Moor J., Nelson J. The Logic Book. 6<sup>th</sup> edition. McGraw-Hill, 2014.*
3. *Gabbay D.M., Rodrigues O.T. Elementary Logic with Application: A Procedural Perspective for Computer Scientists*
4. *Magnus P.D., Button T. forall x: An Introduction to Formal Logic. University of Calgary, 2020.*
5. *Smith P. Introduction to Formal Logic. Cambridge University Press, 2020.*

### **4.2. Supplementary Literature:**

1. *Ayala-Rincon M., de Moura F.L.C. Applied Logic for Computer Scientists: Computational Deduction and Formal Proofs. Springer, 2017.*
2. *Copi I.M., Cohen C., McMahon K. Introduction to Logic. 14<sup>th</sup> edition. Routledge, 2016.*
3. *Girard J.-Y. The Blind Spot. Lectures on Logic. European Mathematical Society, 2011.*
4. *Halbeisen L., Krapf R. Godel's Theorems and Zermelo's Axioms. A Firm Foundation of Mathematics. Springer Nature, 2020.*
5. *Handbook on Philosophical Logic, 2<sup>nd</sup> ed., Vol. I. ed. by Gabbay D.M., Guentner F. Springer Science, 2001.*
6. *Indrzejczak A. Natural Deduction, Hybrid Systems and Modal Logics. (Trends in Logic, Vol. 30). Springer, 2010.*
7. *Indrzejczak A. Sequents and Trees. An Introduction to the Theory and Applications of Propositional Sequent Calculi. (Studies in Universal Logic). Springer Nature, 2021.*
8. *Nguyen H.T., Walker C.L., Walker E.A. A First Course in Fuzzy Logic. 4th edition. CRC Press, 2019.*
9. *Priest G. One. Being and Investigation into the Unity of Reality and of its Parts, including the Singular Object which is Nothingness, 1<sup>st</sup> ed. Oxford University Press, 2014.*
10. *Yaqub A.M. An Introduction to Logical Theory. Broadview Press, 2013.*

**5. Methodological and Thematical Components of the Course**

| № 3/II | <p align="center"><b>List of the Lecture Plans according to the Units<br/>(basic literature, supplementary literature, individual assignments)</b></p>   |
|--------|--|
| 1      | <p><i>Lecture 1. Logic, its Definitions, Applications and Subject. Patterns of Inference</i></p>   |
|        | <p>1. Knowledge and belief.<br/>2. General definitions of logic.<br/>3. Logic and language.<br/>4. Patterns of inference (deduction, induction and abduction).</p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p>   |
| 2.     | <p><i>Lecture 2. Classical Propositional Logic</i></p> <p>1. Historical review of logic development.<br/>2. Classical propositional logic.<br/>3. Concept of logical argument and its compounds.<br/>4. Syllogisms as the building blocks of classical logic.</p> <p>Basic literature: 3, 4.<br/>Supplementary: 1, 9.</p>  |
| 3.     | <p><i>Lecture 3. Syntax of Classical Propositional Logic and Truth Tables basics</i></p> <p>1. Syntax of formal (symbolic) logic.<br/>2. Contingencies, tautologies and contradictions<br/>3. Decidability, satisfiability and Truth Tables basics.<br/>4. Functional completeness of logical connectives.</p> <p>Basic literature: 1, 2, 4.<br/>Supplementary: 1, 3, 6, 10.</p> |
| 4.     | <p><i>Lecture 4. Formal Proofs, Direct and Indirect</i></p> <p>1. Algebraic properties of classical propositional logics.<br/>2. The concept of Formal proof: general notions and aspects.<br/>3. Direct proofs.<br/>4. Indirect proofs.</p> <p>Basic literature: 2, 4, 5.<br/>Supplementary: 2, 10.</p>   |
| 5.     | <p><i>Lecture 5. Axiomatic Deductive Systems</i></p> <p>1. Axiomatic deductive systems.<br/>2. Generalized properties of axiomatic deductive systems.<br/>3. Why axiomatic deductive systems are problematic?<br/>4. Limits of Classical propositional logics.</p> <p>Basic literature: 3, 4,<br/>Supplementary: 1, 2, 6, 7</p>  |
| 6.     | <p><i>Lecture 6. First-Order Predicate Logic family</i></p>  |

|    |  |
|----|--|
|    | <ol style="list-style-type: none"> <li>1. <i>First-Order Predicate Logic: syntax and key concepts.</i></li> <li>2. <i>Universe of Discourse and First-Order Frames.</i></li> <li>3. <i>Key properties of first-order logics family.</i></li> <li>4. <i>Identity versus Equivalence.</i></li> </ol> <p>Basic literature: 1, 2, 5.<br/>Supplementary: 5, 7, 9.</p>   |
| 7. | <p><i>Lecture 7. Natural Deduction Systems of Proof</i></p> <ol style="list-style-type: none"> <li>1. <i>Introduction to the Natural deduction systems.</i></li> <li>2. <i>Rules of Inference: Propositional Logic.</i></li> <li>3. <i>Rules of Inference: First-Order Predicate (Polyadic) Logic.</i></li> <li>4. <i>Derived Rules of Inference.</i></li> </ol> <p>Basic literature: 4, 3.<br/>Supplementary: 5, 7, 10.</p>   |
| 8. | <p><i>Lecture 8. Tree Tableaux as a Decidable system of Proof in CPL and FOPL</i></p> <ol style="list-style-type: none"> <li>1. <i>Properties of Natural Deduction Systems.</i></li> <li>2. <i>Indirect Proofs II.</i></li> <li>3. <i>Truth Trees (Tree Tableaux): proof systems behind Natural Deduction, Truth Tables et. al.</i></li> <li>4. <i>Rules of Decomposition and Tableaux in use.</i></li> </ol> <p>Basic literature: 1, 4.<br/>Supplementary: 2, 6, 9.</p>   |
| 9. | <p><i>Lecture 9. Rule-Based Systems, Semantic Frames, Expert Systems. Beyond the First-Order Logics</i></p> <ol style="list-style-type: none"> <li>1. <i>Limitations and Setbacks of First-Order Predicate Logics.</i></li> <li>2. <i>Rule-Based Systems: Backward- and Forward-Chained Reasoning Systems.</i></li> <li>3. <i>Formal Layout of Forward and Backward Reasoning.</i></li> <li>4. <i>Semantic Networks and Expert Systems of Knowledge Representation.</i></li> <li>5. <i>Computation VS. Deduction.</i></li> </ol> <p>Basic literature: 2, 3, 4, 5.<br/>Supplementary: 1, 2, 5, 8, 10.</p> |

*Lectures with use of interactive education technologies and additional supplementary to each of the topics which are connected with application of the obtained knowledge. The full list of methods includes:*

- Interactive technologies and distance learning tools (visual demonstration of theoretic notions);
- Exercises, seminary discussions and blitz-test to sum up
- Heuristic method of study;
- Case-studies (by examples and counterexamples);
- Theoretical assignments and exercises
- Providing additional special featured sources from selected addendum of foreign authors' academic researches on the given topics (in addition to the lectures)

| № 3/Π | <b>Key Topics and Subjects to Practical classes and Individual assignments</b>  |
|-------|---|
| 1     | <p><b><i>Topic 1. Language, Truth and Logic</i></b></p> <p><i>Knowledge, Perception and Belief. Formal and Informal Reasoning. Induction, Abduction and Deduction as Reasoning ABCs. Logic as a study of structure of human intelligence.</i></p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p> |

|    |   |
|----|---|
| 2. | <p><b>Topic 2. Propositional Logic: Syntax and Concepts</b></p> <p><i>Formal Logic: a Brief Historical Review. The Rise of Symbolic Logic. Classical Propositional Logic. Logical Connectives. CPL: Basic Concepts, Key Notions and Definitions. Proof Theory I: Truth Tables</i></p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p>   |
| 3  | <p><b>Topic 3. Systems of Proof in Propositional Logic: Axiomatic and Natural Deduction Systems</b></p> <p><i>PL Semantics: Interpretation, Valuation, Truth Assignment. 'Laws of Logic' aka Rules of Transformation. The Art of Proof I: Axiomatic Systems. The Art of Proof II: Natural Deduction Systems. 'Fitch-style' (2-column) Proofs. Rules of Inference for NDS.</i></p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p>       |
| 4  | <p><b>Topic 4. First-Order Predicate Logic: Syntax, Concepts, Rules of Inference</b></p> <p><i>Constraints and Limitations of CPL. Expanding the Vocabulary: FOPL Syntax Intro. Individual Constants and Variables. Predicates: unary and n-ary (Properties and Relations). Quantifiers and Identity. Adding Rules for Quantifiers. Monadic Predicate Logic as a FOPL Fragment.</i></p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p> |
| 5. | <p><b>Topic 5. F.O.P.L.: Frames and Rules for Identity operator</b></p> <p><i>First-Order Structures: Interpretations and Models. Domain, Universe of Discourse, Model Building. Identity Rules; Leibniz Law. Proofs with Identity Operator. Soundness and Completeness. Decidability Problem.</i></p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p>  |
| 6. | <p><b>Topic 6. Proof Theory: Additional Notions</b></p> <p><i>Direct Proofs: Conditional Assumption, Countermodel, Pigeonhole Principle, Mathematical Induction etc. Indirect Proofs: General Idea, Methods, Cases. Expanding Proof Techniques: Semantic (Tree) Tableaux. Rules of Decomposition: CPL. Rules of Decomposition: FOPL and Identity. Deduction Theorem.</i></p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p>            |
| 7. | <p><b>Topic 7. Many-Valued Logics</b></p> <p><i>The General Idea behind 3-valuedness (Kleene, Lukasiewicz, Bochvar etc.). Truth Matrices and Valuations with <math>2 &lt; \text{values}</math>. Axiomatic CPL systems for many-valued logics. Deductive Systems for many-valued logics. Fuzzy Logic and Lambda-Calculi. First Degree Entailment.</i></p> <p>Basic literature: 1, 2.<br/>Supplementary: 5, 8.</p>                                |
| 8. | <p><b>Topic 8. Non-Classical Modal Logics</b></p> <p><i>Normal Modal Logics Family. Possible Worlds Semantics. Modal Predicate Logic. Temporal and Epistemic Logics. Expanding Proof Techniques: Sequent Calculus.</i></p> <p>Basic literature: 1, 2.</p>   |

|    |   |
|----|---|
|    | Supplementary: 5, 8.  |
| 9. | <b>Topic 9. Artificialization of Reasoning, AI, SAT etc.</b>  |
|    | <i>From Turing Test to Chinese Room. Automated Theorem Proving: SAT Problem. Logic Programming and other Implementations. Towards 'Mind – Body Problem'.</i><br><br>Basic literature: 1, 2.<br>Supplementary: 5, 8. |

## List of Theoretical Questions for the Practical Classes

### Seminar 1.

1. Knowledge, Belief, Opinion
2. Demarcation Problem
3. Verification and Justification
4. Logic and Epistemology
5. Inductive Reasoning
6. Deductive Reasoning
7. Abductive Reasoning

*Textbooks:* 2, 4, 5.

*Supplementary:* 4, 6, 10.

### Seminar 2.

1. Historical Review of Classical Logic
2. Classical Propositional Logic: Syntax
3. Classical Propositional Logic: Functional Completeness
4. Truth Tables (Proof Techniques I)
5. Valid and Invalid Arguments
6. Truth Functions
7. Formal and Informal Proof

*Textbooks:* 1, 3, 4.

*Supplementary:* 2, 3, 5.

### Seminar 3.

1. CPL Semantics: Models and Countermodels
2. Logical Entailment.
3. Equivalence relations in CPL
4. Consistency and Inconsistency
5. Axiomatic Systems of Proof
6. Natural Deduction Systems
7. Laws of Logic
8. Rules of Inference for NDS

*Textbooks:* 1, 2.

*Supplementary:* 2, 7.

### Seminar 4.

1. *Proof Techniques II: Two-Column 'Fitch' Proof.*
2. Interpretations, Valuations, Truth Assignments

3. Soundness and Completeness
4. Proving (In-/)Consistency with TT
5. Checking (In-/)Validity of Argument with TT
6. Checking Logical Entailment with TT
7. Using TT for Model and Countermodel Building

*Textbooks:* 1-4.

*Supplementary:* 3, 7-10.

### **Seminar 5.**

1. Deduction Theorem
2. CPL Limits
2. Monadic QL Syntax (Unary Predicates, Constants, Variables, Domain)
4. Monadic QL Syntax: Quantifiers
5. First Order Predicate Logic: n-ary (Relational) Predicates
6. Equivalence and Identity Relations
7. FOPL Rules of Inference

*Textbooks:* 1, 4, 5.

*Supplementary:* 2, 3, 9.

### **Seminar 6.**

1. Rules of Inference for FOPL with Identity
2. Decidability Problem I: CPL is Decidable
3. Decidability Problem II: FOPL is Semi-Decidable
4. Satisfiability Problem
5. Proof Techniques III: Truth Trees aka Tableaux

*Textbooks:* 4, 5.

*Supplementary:* 2, 6, 8.

### **Seminar 7.**

1. Direct and Indirect Proofs
2. Conditional Proofs
3. Proof by Exhaustion
4. Case Proofs
5. Countermodel Building Proof
6. Pigeonhole Principle
7. (Strong, Weak, Peano) Mathematical Induction Proofs
8. Indirect Proofs: RAA and Contradiction
9. Indirect Proofs: Non-Sequitur and Contrapositives

*Textbooks:* 1, 2, 3, 5.

*Supplementary:* 4, 7, 8.

### **Seminar 8.**

1. Proof Techniques IV: Sequent Calculus
2. Logical Matrices: Assigning more than 2-valued Truth Values
3. Three-Valued Logics: L3
4. Three-Valued Logics: B3
5. Three-Valued Logics: KS3 and KW3
6. Three-Valued Logics: Pi3



7. Four-Valued (First Degree Entailment) Family of Logics: from Nuel Belnap to Graham Priest
8. n-valued Logics: G5-Gn (Godel's Logics), Post's infinitely-valued
9. n-valued Logics: Post's infinitely-valued Logic

*Textbooks:* 3, 4.

*Supplementary:* 4, 5, 7, 10.

## **Seminar 9.**

1. Proof Techniques V: Resolution Method
2. Normal Modal Propositional Logics
3. Possible Worlds Semantics
4. Modal FOPL's
5. Epistemic, Temporal and Dynamic Logics
6. Automated Reasoning
7. AI r&d and Applied Logic

*Textbooks:* 2, 3, 5.

*Supplementary:* 1, 2, 6, 9, 10.

## **6. Individual Tasks**

*Preparation for the theoretical (oral) answers (presentations) for practical classes (optional, up to the classes dedicated to the topics); solving the written tasks (from 5 to 7 days since the task's form publication); express-tests (optional: from 2 to 8 academic hours);*

Topic 1 Individual tasks: *(1) Think of the cases where Inductive and Abductive inferential patterns are more preferable than Deductive inferences. Give your own examples of cases where Deductive inference fails.*

*(2) Give your own instances for each epistemic state: knowledge, belief and opinion (in informal layout).*

Topic 2 Individual tasks: *(1) Specify (with your own examples and instances) the difference between logical truth and material (or factual) truth, their metaphysical and epistemic asymmetry and possibility of synchrony of both.*

*(2) Construct your own instances and examples of classical syllogisms of Aristotelian syllogistic logic with each figure and mood, basing on the classical square of categorical syllogisms and the table (diagram) from the Methodological recommendations.*

Topic 3 Individual tasks: *(1) Build up three truth tables for three well-formed formulae of Classical propositional logic such, that one of them is a tautology, the second is a contradiction and the third is a contingent sentence. Use no less than two propositional variables and three different logical connectives per formula at a time.*

*(2) Give an example of functionally incomplete set of logical connectives and explain why such a set would be inappropriate for classical propositional logic (or, in other words, what would make it incomplete).*

Topic 4 Individual tasks: *(1) Take four algebraic properties (a.k.a. Rules of Transformation) of Classical Propositional Logic and make four transformations of well-formed formulae of your own composition to demonstrate how these transformation algorithms (their formal schemata laid out in lecture) work practically.*

*(2) Perform such formal proofs as: proof by cases, proof by exhaustion, proof by contradiction and informal (!) proof by non-consequence (on the basis of theoretical material of the lecture and Methodological recommendations).*

Topic 5 Individual tasks: *(1) Demonstrate the differences between the operators of Identity and Equivalence. Specify their formal properties and give examples in formal language and informal*

*specifications and interpretations to demonstrate the philosophical underpinnings of both and why should they be differentiated.*

*(2) Compare Natural Deduction and Axiomatic Systems. What are the weaknesses and advantages of both? Which would you prefer to use, given an option to choose and why?*

*Topic 6 Individual tasks: (1) Construct your own FOPL semantic Frame. Use the Examples added to Methodological recommendations to make sense of it.*

*(2) Think of the reason why polyadic predicate logics are, at most, semi-decidable, or, as it is usual, undecidable at all. What makes them such a problem? Can it be solved in future? What do we need for such a solution to come at all?*

*Topic 7 Individual tasks: (1) By analogy (from the paragraph on CPL limitations) think of any problems and restrictions to which FOPL may be exposed in a similar way.*

*(2) Perform a proof of valid and invalid arguments (with no less than one premise per each of them) of First-Order Predicate Logic in Natural Deductive System of your choosing.*

*Topic 8 Individual tasks: (1) Compare three proof systems: Natural Deduction System; Axiomatic Deductive System; Tree Tableaux. What are their principal differences and similarities? What are their Advantages and disadvantages?*

*(2) Perform two proofs using the Tree Tableaux: one for Classical propositional logic, two – for First-Order Predicate Logic. Use the Rules of Decomposition of both alpha- and beta-groups of Rules.*

*Topic 9 Individual tasks: (1) Specify the challenges for the automated reasoning that hasn't been solved yet. Think of those which may be unresolvable in principle, as such.*

*(2) Give a formal layout of Rule-Governed automated system of proof of your choosing (deductive-pattern governed or computational-pattern governed).*

## **Educational Policy and Assessment**

### **7. Educational policy**

- *Attending Policy: short theses; discussions between the attendants concerning proposed topics or the topics prepared by students themselves previously, visual representations of theses, answering the 'ad-hoc questions' to demonstrate the level of comprehension of the prepared material (from 1 to 3, put by the lecturer after listening to the students answer, concerning the answer contents); due to the contemporary educational paradigm, the communication basics include Google-Meet (for the classes conduction), Zoom, Classroom (for the online but not live communication), G-Disk, Google Sheets and Google Slides. For the lectures provision, the lecturer uses Keynote (synced with ApplePencil + iPad for the live additions to the slides) and, partially, the content of padOS-compatible software, namely: Nyaya; Logic 100; GoodNotes; Wolfram Discrete Math Assistant; Bool-it!; Truth Table; Erpestolz (either purchased by lecturer or used under Creative Commons License).*
- *Individual Tasks: theoretical (oral) questions as-individual-tasks are presented as a short theses/reports concerning the topics and the question, using the main and supplementary literature; the prepared material is desired (but not obligatory) to be supplied with visual representations (diagrams, illustrations, graphs, tables – in the form of presentations);*
- *Additional/Penalty Points Policy: additional points may be obtained for the following forms of individual and classroom activities: putting the questions to other students concerning their answers; participation in discussions or their moderation; completing successfully additional classroom tasks of higher complexity level (proposed by lecturer); completing the individual additional tasks which need the processing of supplementary materials and search of additional information by students themselves and preparation of theoretical questions of advanced level (provided as an addition to the list of planned compulsory questions for practicals);*

## 8. Forms of assessment and facilitating scale evaluation

*Formative assessment:* Tasks performance valuation, consulting on preliminaries, evaluating oral and written proceedings; Preparing theses or presentations, completing exercises

*Border control:* blitz-tests and providing additional special assignment in a form of exercises with a deadline synced to the attesting assessment.

*Summative assessment:* Knowledge Survey

*Summative assessment access conditions:* at least one blitz-test rated  $20 <$  points, and at least two successfully passed written or theoretical individual tasks.

| Rating in points             | Rating   |
|------------------------------|--|
| 100-95                       | Perfectly  |
| 94-85                        | Very Good  |
| 84-75                        | Good   |
| 74-65                        | Satisfactorily                                       |
| 64-60                        | Enough   |
| $> 60$                       | Unsatisfactorily with the possibility of re-assembly |
| Failed to obtain (SA) access | Unsatisfactorily with the obligatory repeated course |

## 9. Appendix

- List of examples of Practical Exercises (Tasks) provided for the Formative Assessment (during the course delivery):

1. Determine, which of the given CPL formulae is a: (a) tautology; (b) contingent sentence; (c) contradiction:

- $a \uparrow (\neg b \vee c)$
- $(a \wedge d) \rightarrow ((c \wedge \neg b) \rightarrow e)$
- $p \wedge (r \leftrightarrow (\neg(\neg p \vee \neg r)))$
- $((n \rightarrow m) \wedge (m \rightarrow n)) \leftrightarrow (n \leftrightarrow m)$
- $(b \vee \neg(a \vee b)) \leftrightarrow \neg b$
- $(n \leftrightarrow (n \wedge \neg n)) \leftrightarrow \neg n$
- $\neg(\perp \leftrightarrow \neg \top)$
- $(p \wedge \neg p) \leftrightarrow \top$
- $((m \vee \neg n) \downarrow (a \oplus \neg b)) \rightarrow (c \leftrightarrow n)$

2. Check the Validity of the arguments by giving the Model with demonstration of Logical Entailment of true Conclusion on any assignment. Use Truth Tables technique.

- $\{\neg A, B \vee C\} \models A$
- $\{A \rightarrow C, D \vee \neg B\} \models B \vee \neg A$
- $\{\neg(A \wedge B) \vee (C \rightarrow \neg D), \neg D \leftrightarrow \neg C, C\} \models (\neg A \wedge D) \vee \neg(B \vee C)$
- $\{A, C \wedge D, \neg(D \vee B)\} \models D \wedge (A \vee \neg A)$

3. Given the set of formulae, check, whether it is consistent (= at least one truth-assignment of the propositional variables maps all the formulae to the 'true' truth value at the same time under the same interpretation):

- $P \vee ((\neg P \wedge Q) \rightarrow R)$
- $\neg(S \vee \neg Q) \wedge \neg(\neg R \leftrightarrow P)$
- $\neg P \rightarrow ((S \vee R) \leftrightarrow \neg R)$
- $\neg(Q \wedge (R \leftrightarrow (\neg P \vee S)))$

4. Given these pairs of formulae of CPL, which of them are logically equivalent to each other (= show the same truth values under the same interpretation)?

(a): 1)  $(p \rightarrow q) \rightarrow (q \rightarrow r)$ ; 2)  $\neg(p \wedge q) \vee r$ ;

(b): 1)  $q \wedge (\neg p \leftrightarrow r)$ ; 2)  $((\neg r \wedge (r \rightarrow p)) \rightarrow \neg q) \rightarrow q$

5. Perform evaluation of the formula in the following models with detailed justification for each subformula truth-assignment. Use 'Fitch-style' technique and apply 'V-rules' with specified references to the corresponding lines of notation.

$v(a) = 1$   
 $v(b) = 0$   
 $v(c) = 0$   
 $v(d) = 1$   
 $v(e) = 0$

$v[((\neg e \wedge (b \vee c)) \rightarrow (a \leftrightarrow \neg b)) \neg(d \leftrightarrow (\neg d \wedge (\neg a \rightarrow e))) \rightarrow ((c \leftrightarrow \neg c) \wedge \neg(\neg d \vee (b \wedge \neg c)))] = ?$

$v(a) = 0$   
 $v(b) = 1$   
 $v(c) = 0$   
 $v(d) = 0$   
 $v(e) = 0$

$v[((\neg e \wedge (b \vee c)) \rightarrow (a \leftrightarrow \neg b)) \neg(d \leftrightarrow (\neg d \wedge (\neg a \rightarrow e))) \rightarrow ((c \leftrightarrow \neg c) \wedge \neg(\neg d \vee (b \wedge \neg c)))] = ?$

$v(a) = 1$   
 $v(b) = 1$   
 $v(c) = 0$   
 $v(d) = 1$   
 $v(e) = 0$

$v[((\neg e \wedge (b \vee c)) \rightarrow (a \leftrightarrow \neg b)) \neg(d \leftrightarrow (\neg d \wedge (\neg a \rightarrow e))) \rightarrow ((c \leftrightarrow \neg c) \wedge \neg(\neg d \vee (b \wedge \neg c)))] = ?$

6. Give a Proof of Validity for the Argument below in Natural Deduction proof-style (use ONE of the sets of Rules of Inference and Transformation Rules (at Your choice) and the Proof Strategy, presented on Lectures 3-6).

1.  $\forall x \forall y (x \neq y)$   
 2.  $\exists x \exists y ((Rya \wedge \neg Fy) \wedge \neg(Gbx \vee \forall z Fz))$   
 $\therefore \exists x \forall y \exists z ((Ryz \wedge Gzy) \wedge ((Fy \wedge \neg Fz) \wedge \neg(Fx \wedge Fy)))$

7. Prove the given theorems using Trees Decomposition Method.

(T1):  $\forall x \exists y [\neg((Hxa \vee Fya) \rightarrow Hxa) \rightarrow Fxy]$   
 (T2):  $(Pa \wedge (a = b)) \rightarrow ((b = c) \rightarrow Pc)$   
 (T3):  $\forall x \forall y \forall z (((x=y) \wedge (y=z)) \rightarrow (x=z))$

8. Which of the FOPL sentences would be true under the following interpretation for the given model M?

D: {A, B, C, D};

a: A  
 b: A  
 c: D  
 d: C

F: {<a>, <c>, <d>};  
 G: {<a, a>, <b, a>, <b, d>, <c, b>, <c, a>};

H: {<d, c>, <d, a>, <a, d>, <b, c>, <c, c>};

- (1)  $\forall x \neg Fx$ ;
- (2)  $Gbc \wedge Gbd$
- (3)  $\exists x \exists y (Hxy \wedge \neg Fyx) \rightarrow \forall z Fz$
- (4)  $Hdd \vee Hcb$
- (5)  $\forall x (\neg Gax \rightarrow (Fx \leftrightarrow Fd))$
- (6)  $\exists x \forall y ((\neg Fx \vee Fy) \rightarrow (Hyx \rightarrow (Gxx \wedge Hcy)))$
- (7)  $(a=b) \wedge \forall x ((x=a) \rightarrow (x=b))$

9. Transform the WFF into Clauses, suitable for the Resolution Procedure, using the ruleset I.N.D.O.:

$(\neg p \wedge q) \vee (s \leftrightarrow r) \rightarrow (p \wedge (\neg s \vee \neg q))$

10. Prove a Theorem ' $((r \leftrightarrow (p \wedge (\neg p \wedge q))) \vee (\neg q \wedge s)) \leftrightarrow \neg r$ ' by Resolution Method, applied to the following set of Premises:

- { $\neg p, q, \neg r$ } (Premise 1)
- { $s, p, \neg q$ } (Premise 2)
- { $r$ } (Premise 3)

• *Theoretical Questions for the final Knowledge Survey (Summative Assessment):*

1. Compare and give the stipulative definitions to Knowledge, Belief and Opinion
2. Provide an outline of Demarcation problem
3. Name and compare the paradigmatic patterns of reasoning
4. Give the definition, enumerate key features of Deductive reasoning and deductive argument (with examples)
5. Give the definition, enumerate key features of Inductive reasoning and deductive argument (with examples)
6. Give the definition, enumerate key features of Abductive reasoning and deductive argument (with examples)
7. Compare the procedures of Validation, Justification and Verification. Clarify the relations between them.
8. Define the key components for the Syntax in Classical Propositional Logic formal language. Build an example.
9. Build a model for Valuations and Interpretations for Classical Propositional Logic.
10. Describe the proof technique Truth Tables and outline the ways they are used in Logic and beyond.
11. Give the definition of Functional Completeness and compare two or more functionally complete sets of logical connectives.
17. Specify the Formal definitions and demonstrate them for CPL.
18. Name and define full list of 16 logical connectives for Propositional Logic and show how they are derived from 5 essentials.
19. Name the differences between formal and informal Proofs.
20. Give a metanalytical review for the types and kinds of Proof Systems.
21. Compare Axiomatic Systems of Proof and Natural Deduction Systems.
22. Describe and give the examples of Natural Deduction Systems and differences between them.

23. *Prove the Deduction Theorem in Classical Propositional Logic.*
24. *Prove the Soundness Theorem in Classical Propositional Logic.*
25. *Prove the Completeness Theorem in Classical Propositional Logic.*
26. *Prove the Compactness Theorem in Classical Propositional Logic.*
27. *Prove the Deduction Theorem in First-Order Predicate Logic.*
28. *Prove the Soundness Theorem in First-Order Predicate Logic.*
29. *Prove the Completeness Theorem in First-Order Predicate Logic.*
30. *Prove the Compactness Theorem in First-Order Predicate Logic.*
31. *Give an account on Decidability in Classical Propositional Logic.*
32. *Describe the Decidability Problem for Predicate Logic.*
33. *Build a Model for Interpretation and Truth-Assignments for Monadic Predicate Logic*
34. *Build a Model for Interpretation and Truth-Assignments for a full First-Order Predicate Logic*
35. *Specify the Differences between First-Order Predicate Logic with and without Identity Predicate*
36. *Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Classical PL system*
37. *Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for the Monadic Predicate Logic*
38. *Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for the First-Order Predicate Logic (without Identity)*
39. *Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for the First-Order Predicate Logic with Identity*
40. *Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Basic Modal Logic*
41. *Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Normal Modal Propositional Logic*
42. *Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Modal Predicate Logic*
43. *Enumerate and Specify the functions for all the subsets of Rules in Sequential Calculus in Gerhard Gentzen's deduction system.*
44. *Name and compare alpha-, beta- and  $\forall$ -E F.O. Tableaux Rules subsets of the set of Rules of Decomposition*
45. *Compare Backward and Forward Reasoning algorithms*
46. *Define the General Idea behind Many-Valued Logic*
47. *Describe and Compare FDE-(4-valued) Logic, L3 logic, K3w, K3s and G4/5 logics*
48. *Represent the Idea behind Possible Worlds Semantics*
49. *Specify the Differences between First- and Second-Order Predicate Logic*
50. *Specify the differences for the Initial and Derived Rules of Inference in Natural Deduction systems.*

### **Course Syllabus:**

**Programmer:** lecturer, Candidate of philosophical sciences, Mstislav Kazakov

**Approved by** Department of Philosophy (protocol № 21 from 25.05.2023)

**Agreed with** Methodological Council of Igor Sikorsky Kyiv Polytechnic Institute (protocol № 8 from 02.06.2023)