



LOGIC

Course Syllabus

Course Requisites

Educational degree	<i>First (bachelor's degree)</i>
Knowledge domain	<i>General</i>
Program	<i>General</i>
Educational program	<i>General</i>
Course status	<i>Optional</i>
Form of Education	<i>Full-time/External/Part-Time</i>
Academic year, semester	<i>2nd academic year, 1th and 2nd semesters</i>
Discipline volume	<i>2 credits ECTS (60 hours) / Lectures (18 hrs) / Seminars (18 hrs) / Individual work (24 hrs)</i>
Final assessment/ evaluation methods	<i>Knowledge Survey</i>
Schedule of the Classes	<i>http://rozklad.kpi.ua/Schedules/ScheduleGroupSelection.aspx</i>
Learning Language	<i>English</i>
About the Instructor(s)	<i>Lectures: candidate of philosophical sciences, lecturer, Mstislav Kazakov (mstkazakov@gmail.com; +380930562957) Practical classes: candidate of philosophical sciences, lecturer, Mstislav Kazakov (mstkazakov@gmail.com; +380930562957) the Department of Philosophy Research and Teaching Staff</i>
Course URL	<i>https://classroom.google.com/c/MjU0MjYxODMwODgz</i>

Educational Program

1. Course description, Aims and Scope, Learning outcomes

The **main objective** of the course 'Logic' is an introduction to contemporary areas of research which in manifold extend intersect with laws, algorithms, patterns of human thought, inferencing and reasoning as an attempt to specify the demarcation line between 'sentience' and 'sapience'. According to such a tasks, the main **subject** of the discipline deals with: automated reasoning; natural deduction; Artificial Intelligence (AI) research and development; theory and practice of proofs; formal modelling theory and practice; metamathematics; theories of truth; patterns of human thought (up to some meta-theoretical aspects concerning logic programming and introduction to knowledge bases, expert systems and knowledge representation methods).

The following **program results** are obtained after the familiarization with the course:

- formal representations of the algorithms of human reasoning and inferencing;
- determining the validity of arguments and checking the input data for errors and invalidity;
- performing inductive reasoning procedures in scientific research and professional activity;
- learning theory and using several techniques of formal and informal proofs within the formal systems, such as:
 - 'Gentzen-style' Sequent Calculus;
 - 'Fitch-style' proofs;
- Tableaux methodology;
- Automatic resolution systems etc.;
- improving operating skills concerning the concepts of models for the formal systems;
- calculating the truth-contingence of propositional functions, simple and compound propositions;
- building truth tables for any Boolean formula;
- building Venn's diagrams for the argument validity evaluation;
- building and using logical matrices for many-valued logics;

- operating with the elements of First- and Second-Order Predicate Logics and Modal logics;
- apply rules of inference for theoretical (reasoning) and practical tasks;
- decipher symbolic expressions / modeling semantics for the interpretation of Propositional Functions and Compound Propositions.

2. Prerequisites and Corerequisites of a Course

The course is optional and, while being oriented on polytechnic profiles, is accessible to any bachelor degree program enrollees. That is, no special knowledge is required. Among the desirable (but not obligatory) skills and knowledge there may be mentioned basic understanding of Set Theory, pre-calculus, mathematical proofs basics, automata theory basics and insights into semantics. The recommended level of English for the successful comprehension is B1.

The course is organized and structured in soundness with the paradigm of polytechnical education, having in mind the inclusion of truly useful parts of vast area of logical investigations for this paradigm. The knowledge and practical assignments are intended to serve as one of the access points (of introduction) to such areas of further professional education as Data Science, Fuzzy sets and Automata theory, Expert systems, AI Research, Machine reasoning, Machine Learning, cognitive sciences areas, Neuron Networks and Consciousness applied studies withing Mind-Modelling Paradigm. This is only the list of paradigmatic disciplines, not mentioning the secondary, to which the course 'Logic' may serve as introduction because of wide area of applications of formal logic in contemporary scientific and technological investigations, which is constantly increasing.

3. Course Summary

PART I. CLASSICAL PROPOSITIONAL LOGIC

A. PATTERNS OF REASONING

1. Knowledge, Belief, Opinion. Demarcation Problem.
2. Patterns of Reasoning. Arguments and Propositions. Logic as a Study of Thought.
3. 'Laws of Thought': the Rise of Propositional Logic. Rules of Transformation.

B. FORMAL LOGIC: THE ESSENTIALS

4. Introduction to Classical Propositional Logic. Syntax, Semantics, Valuations. Truth Tables.
5. Consistency, Soundness, Completeness, Validity. Logical Entailment.

C. INSIGHTS INTO PROOF THEORY

6. Proof Systems. Logic as a Proof Theory. Formal System: the Intro. Rules of Inference.

PART II. FIRST ORDER PREDICATE LOGIC

A. PROOF THEORY: ADVANCED TOPICS

7. Axiomatic Systems. Logic as Metamathematics. Logicism and Formalism. Intuitionism and Constructivism. Rules of Inference, Axioms, Axiom Schemata, Theorems in FS.

8. Natural Deduction Systems of Proof: Two-Column Proofs (Fitch, Lemmon, Suppes); Sequent Calculus;

B. BEYOND THE PROPOSITIONAL CALCULI

9. First Order Predicate Logic. Constants, Variables, Quantifiers, Unary Predicates (the Fragment 'Monadic PL').
10. Relations and Functions: n -ary predicates. Domain as a key notion of FOPL.
11. Model Theory: Interpretations, Valuations and Semantics with Quantifiers.
12. FOPL: Inference Rules and Equivalences.
13. Quantification with Identity. Rules and Proofs. Identity Theory ABCs.

C: DECIDABILITY PROBLEM

14. Proof Theory Expanded: Tree Tableaux Method.
15. Decidability Problem. 'FOPL is semi-decidable'. CPL decidability.
16. Resolution Procedure
17. Godel's Proofs of Incompleteness.

PART III. NON-CLASSICAL LOGICS

A. MODAL LOGIC AND POSSIBLE WORLDS

18. Normal Modal Logics: Classical, Epistemic, Linear Temporal PL,
19. Deontic, Temporal and Alethic Logics.
20. Quantification of Modal Logic.
21. Kripke Semantics of Possible worlds.

B. MANY-VALUEDNESS

22. 3-valued logics: the Idea. (L3, B3, Kw3, Ks3, Pi3, G3)
23. FDE Logics Family (Belnap, Steele, Priest, G4)

24. *n-valuedness. Alternatives to more-than-2-valuedness.*

25. *Fuzzy Sets and Fuzzy Logic.*

C. APPLIED LOGIC AND BEYOND

26. *Representing the Knowledge: Fact, Data, Knowledge Bases.*

27. *Use of FOPL in Expert Systems and Semantic Frames etc.*

28. *Automated Reasoning. Resolution Method. Provers of Theorems. SAT.*

29. *Formal systems in Technology, Programming, Applied sciences and beyond.*

4. Course Texts and/or Other Study Materials

4.1. Course Textbooks:

1. Barker-Plummer D., Barwise J., Etchemendy J. *Language, Proof and Logic*. 2nd edition. CSLI Publications, 2011.

2. Bergmann M., Moor J., Nelson J. *The Logic Book*. 6th edition. McGraw-Hill, 2014.

3. Gabbay D.M., Rodrigues O.T. *Elementary Logic with Application: A Procedural Perspective for Computer Scientists*

4. Magnus P.D., Button T. *forall x: An Introduction to Formal Logic*. University of Calgary, 2020.

5. Smith P. *Introduction to Formal Logic*. Cambridge University Press, 2020.

4.2. Supplementary Literature:

1. Ayala-Rincon M., de Moura F.L.C. *Applied Logic for Computer Scientists: Computational Deduction and Formal Proofs*. Springer, 2017.

2. Copi I.M., Cohen C., McMahon K. *Introduction to Logic*. 14th edition. Routledge, 2016.

3. Girard J.-Y. *The Blind Spot. Lectures on Logic*. European Mathematical Society, 2011.

4. Halbeisen L., Krapf R. *Gödel's Theorems and Zermelo's Axioms. A Firm Foundation of Mathematics*. 5. Springer Nature, 2020.

5. *Handbook on Philosophical Logic*, 2nd ed., Vol. I. ed. by Gabbay D.M., Guenther F. Springer Science, 2001.

6. Indrzejczak A. *Natural Deduction, Hybrid Systems and Modal Logics*. (Trends in Logic, Vol. 30). Springer, 2010.

7. Indrzejczak A. *Sequents and Trees. An Introduction to the Theory and Applications of Propositional Sequent Calculi*. (Studies in Universal Logic). Springer Nature, 2021.

8. Nguyen H.T., Walker C.L., Walker E.A. *A First Course in Fuzzy Logic*. 4th edition. CRC Press, 2019.

9. Priest G. *One. Being and Investigation into the Unity of Reality and of its Parts, including the Singular Object which is Nothingness*, 1st ed. Oxford University Press, 2014.

10. Yaqub A.M. *An Introduction to Logical Theory*. Broadview Press, 2013.

Educational Content

5. Methodological and Thematical Components of the Course

Lectures

Lecture 1. Patterns of Reasoning. Aims and Scope of Logical Sciences

1. Knowledge and Belief.

2. Demarcation Problem.

3. Formal and Informal Reasoning.

4. Induction, Abduction and Deduction as Reasoning ABCs.

Basic literature: 1, 2,

Supplementary: 5, 8

Individual task: Think of the cases where Inductive and Abductive inferential patterns are more preferable than Deductive inferences. Give your own examples of cases where Deductive inference fails.

Lecture 2. Classical Propositional Logic: Syntax and Essentials

1. Formal Logic: a Brief Historical Review.

2. The Rise of Symbolic Logic.

3. Classical Propositional Logic.

4. Logical Connectives.

5. CPL: Basic Concepts, Key Notions and Definitions.

6. Proof Theory I: Truth Tables

Basic literature: 3, 4,

Supplementary: 1, 9

Lecture 3. Natural Deduction and CPL Semantics

1. *PL Semantics: Interpretation, Valuation, Truth Assignment.*
2. *'Laws of Logic' aka Rules of Transformation.*
3. *The Art of Proof I: Axiomatic Systems.*
4. *The Art of Proof II: Natural Deduction Systems.*
5. *'Fitch-style' (2-column) Proofs.*
6. *Rules of Inference for NDS.*

Basic literature: 1, 2, 4

Supplementary: 1, 3, 6, 10

Individual task: Compare Natural Deduction and Axiomatic Systems. What are the weaknesses and advantages of both? Which would you prefer to use, given an option to choose and why?

Lecture 4. First-Order Predicate Logic I

1. *Constraints and Limitations of CPL*
2. *Expanding the Vocabulary: FOPL Syntax Intro*
3. *Individual Constants and Variables*
4. *Predicates: unary and n-ary (Properties and Relations)*
5. *Quantifiers and Identity*
6. *Adding Rules for Quantifiers*
7. *Monadic Predicate Logic as a FOPL Fragment.*

Basic literature: 2, 4, 5

Supplementary: 2, 10

Individual task: By analogy (from the paragraph on CPL limitations) think of any problems and restrictions to which FOPL may be exposed in a similar way.

Lecture 5. First-Order Predicate Logic II

1. *First-Order Structures: Interpretations and Models*
2. *Domain, Universe of Discourse, Model Building*
3. *Identity Rules; Leibniz Law*
4. *Proofs with Identity Operator*
5. *Soundness and Completeness*
6. *Decidability Problem*

Basic literature: 3, 4,

Supplementary: 1, 2, 6, 7

Individual task: Construct your own FOPL semantic Frame. Use the Examples added to a Classroom assignment to make sense of it!

Lecture 6. Proof Theory: Advanced Topics

1. *Direct Proofs: Conditional Assumption, Countermodel, Pigeonhole Principle, Mathematical Induction etc.*
2. *Indirect Proofs: General Idea, Methods, Cases*
3. *Expanding Proof Techniques: Semantic (Tree) Tableaux*
4. *Rules of Decomposition: CPL*
5. *Rules of Decomposition: FOPL and Identity*
6. *Deduction Theorem*

Basic literature: 1, 2, 5

Supplementary: 5, 7, 9

Lecture 7. N-valuedness in Propositional and Predicate Logics

1. *The General Idea behind 3-valuedness (Kleene, Lukasiewicz, Bochvar etc.)*
2. *Truth Matrices and Valuations with $2 < v$ values*
3. *Axiomatic CPL systems for many-valued logics*
4. *Deductive Systems for many-valued logics*
5. *Fuzzy Logic and Lambda-Calculi*
6. *First Degree Entailment*

Basic literature: 4, 3,

Supplementary: 5, 7, 10

Individual task: Construct your own n-valued logic. Introduce two connectors and specify their truth values. Give your own account on the set of v-function with designated / undesignated values.

Lecture 8. Non-Classical Logics

1. Normal Modal Logics Family
2. Possible Worlds Semantics
3. Modal Predicate Logic
4. Temporal and Epistemic Logics
5. Expanding Proof Techniques: Sequent Calculus

Basic literature: 1, 4

Supplementary: 2, 6, 9

Individual task: Construct a Frame with two or three possible worlds and a truth table for up to 5 formulae with their evaluation.

Lecture 9. Logic and Artificial Intelligence

1. From Turing Test to Chinese Room
2. Automated Theorem Proving: SAT Problem
3. Logic Programming and other Implementations
4. Towards 'Mind – Body Problem'

Basic literature: 2, 3, 4, 5

Supplementary: 1, 2, 5, 8, 10

Individual task: Specify the challenges for the automated reasoning that hasn't been solved yet. Think of those which may be unresolvable in principle, as such.

Lectures with use of interactive education technologies and additional supplementary to each of the topics which are connected with application of the obtained knowledge. The full list of methods includes:

- Interactive technologies and distance learning tools (visual demonstration of theoretic notions);
- Exercises, seminary discussions and blitz-test to sum up
- Heuristic method of study;
- Case-studies (by examples and counterexamples);
- Theoretical assignments and exercises
- Providing additional special featured sources from selected addendum of foreign authors' academic researches on the given topics (in addition to the lectures)

Practical classes

Topic 1. Language, Truth and Logic

Knowledge, Perception and Belief. Formal and Informal Reasoning. Induction, Abduction and Deduction as Reasoning ABCs. Logic as a study of structure of human intelligence.

Basic literature: 1, 2,

Supplementary: 5, 8

Topic 2. Propositional Logic: Syntax and Concepts

Formal Logic: a Brief Historical Review. The Rise of Symbolic Logic. Classical Propositional Logic. Logical Connectives. CPL: Basic Concepts, Key Notions and Definitions. Proof Theory I: Truth Tables

Basic literature: 1, 2,

Supplementary: 5, 8

Topic 3. Systems of Proof in Propositional Logic: Axiomatic and Natural Deduction Systems

PL Semantics: Interpretation, Valuation, Truth Assignment. 'Laws of Logic' aka Rules of Transformation. The Art of Proof I: Axiomatic Systems. The Art of Proof II: Natural Deduction Systems. 'Fitch-style' (2-column) Proofs. Rules of Inference for NDS.

Basic literature: 1, 2,

Supplementary: 5, 8

Topic 4. First-Order Predicate Logic: Syntax, Concepts, Rules of Inference

Constraints and Limitations of CPL. Expanding the Vocabulary: FOPL Syntax Intro. Individual Constants and Variables. Predicates: unary and n-ary (Properties and Relations). Quantifiers and Identity. Adding Rules for Quantifiers. Monadic Predicate Logic as a FOPL Fragment.

Basic literature: 1, 2,

Supplementary: 5, 8

Topic 5. F.O.P.L.: Frames and Rules for Identity operator

First-Order Structures: Interpretations and Models. Domain, Universe of Discourse, Model Building. Identity Rules; Leibniz Law. Proofs with Identity Operator. Soundness and Completeness. Decidability Problem.

Basic literature: 1, 2,
Supplementary: 5, 8

Topic 6. Proof Theory: Additional Notions

Direct Proofs: Conditional Assumption, Countermodel, Pigeonhole Principle, Mathematical Induction etc. Indirect Proofs: General Idea, Methods, Cases. Expanding Proof Techniques: Semantic (Tree) Tableaux. Rules of Decomposition: CPL. Rules of Decomposition: FOPL and Identity. Deduction Theorem.

Basic literature: 1, 2,
Supplementary: 5, 8

Topic 7. Many-Valued Logics

The General Idea behind 3-valuedness (Kleene, Lukasiewicz, Bochvar etc.). Truth Matrices and Valuations with 2 < values. Axiomatic CPL systems for many-valued logics. Deductive Systems for many-valued logics. Fuzzy Logic and Lambda-Calculi. First Degree Entailment.

Basic literature: 1, 2,
Supplementary: 5, 8

Topic 8. Non-Classical Modal Logics

Normal Modal Logics Family. Possible Worlds Semantics. Modal Predicate Logic. Temporal and Epistemic Logics. Expanding Proof Techniques: Sequent Calculus.

Basic literature: 1, 2,
Supplementary: 5, 8

Topic 9. Artificialization of Reasoning, AI, SAT etc.

From Turing Test to Chinese Room. Automated Theorem Proving: SAT Problem. Logic Programming and other Implementations. Towards 'Mind – Body Problem'.

Basic literature: 1, 2,
Supplementary: 5, 8

List of Theoretical Questions for the Practical Classes

Seminar 1

1. Knowledge, Belief, Opinion
2. Demarcation Problem
3. Verification and Justification
4. Logic and Epistemology
5. Inductive Reasoning
6. Deductive Reasoning
7. Abductive Reasoning

Textbooks: 2, 4, 5

Supplementary: 4, 6, 10

Seminar 2

1. Historical Review of Classical Logic
2. Classical Propositional Logic: Syntax
3. Classical Propositional Logic: Functional Completeness
4. Truth Tables (Proof Techniques I)
5. Valid and Invalid Arguments
6. Truth Functions
7. Formal and Informal Proof

Textbooks: 1, 3, 4

Supplementary: 2, 3, 5

Seminar 3

1. CPL Semantics: Models and Countermodels

2. Logical Entailment.
3. Equivalence relations in CPL
4. Consistency and Inconsistency
5. Axiomatic Systems of Proof
6. Natural Deduction Systems
7. Laws of Logic
8. Rules of Inference for NDS

Textbooks: 1, 2

Supplementary: 2, 7

Seminar 4

1. Proof Techniques II: Two-Column 'Fitch' Proof.
2. Interpretations, Valuations, Truth Assignments
3. Soundness and Completeness
4. Proving (In-/)Consistency with TT
5. Checking (In-/)Validity of Argument with TT
6. Checking Logical Entailment with TT
7. Using TT for Model and Countermodel Building

Textbooks: 1-4

Supplementary: 3, 7-10

Seminar 5

1. Deduction Theorem
2. CPL Limits
2. Monadic QL Syntax (Unary Predicates, Constants, Variables, Domain)
4. Monadic QL Syntax: Quantifiers
5. First Order Predicate Logic: n-ary (Relational) Predicates
6. Equivalence and Identity Relations
7. FOPL Rules of Inference

Textbooks: 1, 4, 5

Supplementary: 2, 3, 9

Seminar 6

1. Rules of Inference for FOPL with Identity
2. Decidability Problem I: CPL is Decidable
3. Decidability Problem II: FOPL is Semi-Decidable
4. Satisfiability Problem
5. Proof Techniques III: Truth Trees aka Tableaux

Textbooks: 4, 5

Supplementary: 2, 6, 8

Seminar 7

1. Direct and Indirect Proofs
2. Conditional Proofs
3. Proof by Exhaustion
4. Case Proofs
5. Countermodel Building Proof
6. Pigeonhole Principle
7. (Strong, Weak, Peano) Mathematical Induction Proofs
8. Indirect Proofs: RAA and Contradiction
9. Indirect Proofs: Non-Sequitur and Contrapositives

Textbooks: 1, 2, 3, 5

Supplementary: 4, 7, 8

Seminar 8

1. Proof Techniques IV: Sequent Calculus
2. Logical Matrices: Assigning more than 2-valued Truth Values
3. Three-Valued Logics: L3
4. Three-Valued Logics: B3

5. Three-Valued Logics: KS3 and KW3
6. Three-Valued Logics: Pi3
7. Four-Valued (First Degree Entailment) Family of Logics: from Nuel Belnap to Graham Priest
8. n-valued Logics: G5-Gn (Godel's Logics), Post's infinitely-valued
9. n-valued Logics: Post's infinitely-valued Logic

Textbooks: 3, 4

Supplementary: 4, 5, 7, 10

Seminar 9

1. Proof Techniques V: Resolution Method
2. Normal Modal Propositional Logics
3. Possible Worlds Semantics
4. Modal FOPL's
5. Epistemic, Temporal and Dynamic Logics
6. Automated Reasoning
7. AI r&d and Applied Logic

Textbooks: 2, 3, 5

Supplementary: 1, 2, 6, 9, 10

6. Individual Tasks

Preparation for the theoretical (oral) answers (presentations) for practical classes (optional, up to the classes dedicated to the topics); solving the written tasks (from 5 to 7 days since the task's form publication); express-tests (optional: from 2 to 8 academic hours).

Educational Policy and Assessment

7. Educational policy

- *Attending Policy: short theses; discussions between the attendants concerning proposed topics or the topics prepared by students themselves previously, visual representations of theses, answering the 'ad-hoc questions' to demonstrate the level of comprehension of the prepared material (from 1 to 3, put by the lecturer after listening to the students answer, concerning the answer contents); due to the contemporary educational paradigm, the communication basics include Google-Meet (for the classes conduction), Zoom, Classroom (for the online but not live communication), G-Disk, Google Sheets and Google Slides. For the lectures provision, the lecturer uses Keynote (synced with ApplePencil + IPad for the live additions to the slides) and, partially, the content of padOS-compatible software, namely: Nyaya; Logic 100; GoodNotes; Wolfram Discrete Math Assistant; Bool-it!; Truth Table; Erpestolz (either purchased by lecturer or used under Creative Commons License).*
- *Individual Tasks: theoretical (oral) questions as-individual-tasks are presented as a short theses/reports concerning the topics and the question, using the main and supplementary literature; the prepared material is desired (but not obligatory) to be supplied with visual representations (diagrams, illustrations, graphs, tables – in the form of presentations);*
- *Additional/Penalty Points Policy: additional points may be obtained for the following forms of individual and classroom activities: putting the questions to other students concerning their answers; participation in discussions or their moderation; completing successfully additional classroom tasks of higher complexity level (proposed by lecturer); completing the individual additional tasks which need the processing of supplementary materials and search of additional information by students themselves and preparation of theoretical questions of advanced level (provided as an addition to the list of planned compulsory questions for practicals).*

8. Forms of assessment and facilitating scale evaluation

Formative assessment: Tasks performance valuation, consulting on preliminaries, evaluating oral and written proceedings; Preparing theses or presentations, completing exercises

Border control: blitz-tests and providing additional special assignment in a form of exercises with a deadline synced to the attesting assessment.

Summative assessment: Knowledge Survey

Summative assessment access conditions: at least one blitz-test rated 20< points, and at least two successfully passed written or theoretical individual tasks.

Rating in points	Rating
100-95	Perfectly
94-85	Very Good
84-75	Good
74-65	Satisfactorily
64-60	Enough
> 60	Unsatisfactorily with the possibility of re-assembly
Failed to obtain (SA) access	Unsatisfactorily with the obligatory repeated course

9. Appendix

List of examples of Practical Exercises (Tasks) provided for the Formative Assessment (during the course delivery):

1. Determine, which of the given CPL formulae is a: (a) tautology; (b) contingent sentence; (c) contradiction:

- (1) $a \uparrow (\neg b \vee c)$
- (2) $(a \wedge d) \rightarrow ((c \wedge \neg b) \rightarrow e)$
- (3) $p \wedge (r \leftrightarrow (\neg(\neg p \vee \neg r)))$
- (4) $((n \rightarrow m) \wedge (m \rightarrow n)) \leftrightarrow (n \leftrightarrow m)$
- (5) $(b \vee \neg(a \vee b)) \leftrightarrow \neg b$
- (6) $(n \leftrightarrow (n \wedge \neg n)) \leftrightarrow \neg n$
- (7) $\neg(\perp \leftrightarrow \neg T)$
- (8) $(p \wedge \neg p) \leftrightarrow T$
- (9) $((m \vee \neg n) \downarrow (a \oplus \neg b)) \rightarrow (c \leftrightarrow n)$

2. Check the Validity of the arguments by giving the Model with demonstration of Logical Entailment of true Conclusion on any assignment. Use Truth Tables technique.

- (1) $\{\neg A, B \vee C\} \models A$
- (2) $\{A \rightarrow C, D \vee \neg B\} \models B \vee \neg A$
- (3) $\{\neg(A \wedge B) \vee (C \rightarrow \neg D), \neg D \leftrightarrow \neg C, C\} \models (\neg A \wedge D) \vee \neg(B \vee C)$
- (4) $\{A, C \wedge D, \neg(D \vee B)\} \models D \wedge (A \vee \neg A)$

3. Given the set of formulae, check, whether it is consistent (= at least one truth-assignment of the propositional variables maps all the formulae to the 'true' truth value at the same time under the same interpretation):

- (1) $P \vee ((\neg P \wedge Q) \rightarrow R)$
- (2) $\neg(S \vee \neg Q) \wedge \neg(\neg R \leftrightarrow P)$
- (3) $\neg P \rightarrow ((S \vee R) \leftrightarrow \neg R)$
- (4) $\neg(Q \wedge (R \leftrightarrow (\neg P \vee S)))$

4. Given these pairs of formulae of CPL, which of them are logically equivalent to each other (= show the same truth values under the same interpretation)?

- (a): 1) $(p \rightarrow q) \rightarrow (q \rightarrow r)$; 2) $\neg(p \wedge q) \vee r$;
- (b): 1) $q \wedge (\neg p \leftrightarrow r)$; 2) $((\neg r \wedge (r \rightarrow p)) \rightarrow \neg q) \rightarrow q$

5. Perform evaluation of the formula in the following models with detailed justification for each subformula truth-assignment. Use 'Fitch-style' technique and apply 'V-rules' with specified references to the corresponding lines of notation.

- $v(a) = 1$
 $v(b) = 0$
 $v(c) = 0$
 $v(d) = 1$
 $v(e) = 0$

$$v[((\neg e \wedge (b \vee c)) \rightarrow (a \leftrightarrow \neg b)) \neg(d \leftrightarrow (\neg d \wedge (\neg a \rightarrow e))) \rightarrow ((c \leftrightarrow \neg c) \wedge \neg(\neg d \vee (b \wedge \neg c)))] = ?$$

$v(a) = 0$

$v(b) = 1$
 $v(c) = 0$
 $v(d) = 0$
 $v(e) = 0$

$v[((\neg e \wedge (b \vee c)) \rightarrow (a \leftrightarrow \neg b)) \neg(d \leftrightarrow (\neg d \wedge (\neg a \rightarrow e))) \rightarrow ((c \leftrightarrow \neg c) \wedge \neg(\neg d \vee (b \wedge \neg c)))] = ?$

$v(a) = 1$
 $v(b) = 1$
 $v(c) = 0$
 $v(d) = 1$
 $v(e) = 0$

$v[((\neg e \wedge (b \vee c)) \rightarrow (a \leftrightarrow \neg b)) \neg(d \leftrightarrow (\neg d \wedge (\neg a \rightarrow e))) \rightarrow ((c \leftrightarrow \neg c) \wedge \neg(\neg d \vee (b \wedge \neg c)))] = ?$

6. Give a Proof of Validity for the Argument below in Natural Deduction proof-style (use ONE of the sets of Rules of Inference and Transformation Rules (at Your choice) and the Proof Strategy, presented on Lectures 3-6).

1. $\forall x \forall y (x \neq y)$
2. $\exists x \exists y ((Ryx \wedge \neg Fy) \wedge \neg(Gbx \vee \forall z Fz))$

$\therefore \exists x \forall y \exists z ((Ryz \wedge Gzy) \wedge ((Fy \wedge \neg Fz) \wedge \neg(Fx \wedge Fy)))$

7. Prove the given theorems using Trees Decomposition Method.

- (T1): $\forall x \exists y [\neg((Hxa \vee Fya) \rightarrow Hxa) \rightarrow Fxy]$
(T2): $(Pa \wedge (a = b)) \rightarrow ((b = c) \rightarrow Pc)$
(T3): $\forall x \forall y \forall z ((x=y) \wedge (y=z)) \rightarrow (x=z)$

8. Which of the FOPL sentences would be true under the following interpretation for the given model M?
D: {A, B, C, D};

a: A
b: A
c: D
d: C

F: {<a>, <c>, <d>};
G: {<a, a>, <b, a>, <b, d>, <c, b>, <c, a>};
H: {<d, c>, <d, a>, <a, d>, <b, c>, <c, c>};

- (1) $\forall x \neg Fx$;
- (2) $Gbc \wedge Gbd$
- (3) $\exists x \exists y (Hxy \wedge \neg Fyx) \rightarrow \forall z Fz$
- (4) $Hdd \vee Hcb$
- (5) $\forall x (\neg Gax \rightarrow (Fx \leftrightarrow Fd))$
- (6) $\exists x \forall y ((\neg Fx \vee Fy) \rightarrow (Hyx \rightarrow (Gxx \wedge Hcy)))$
- (7) $(a=b) \wedge \forall x ((x=a) \rightarrow (x=b))$

9. Transform the WFF into Clauses, suitable for the Resolution Procedure, using the ruleset I.N.D.O.:
 $(\neg p \wedge q) \vee (s \leftrightarrow r) \rightarrow (p \wedge (\neg s \vee \neg q))$

10. Prove a Theorem $((r \leftrightarrow (p \wedge (\neg p \wedge q))) \vee (\neg q \wedge s)) \leftrightarrow \neg r'$ by Resolution Method, applied to the following set of Premises:

- $\{\neg p, q, \neg r\}$ (Premise 1)
 $\{s, p, \neg q\}$ (Premise 2)
 $\{r\}$ (Premise 3)

Theoretical Questions for the final Knowledge Survey (Summative Assessment):

1. Compare and give the stipulative definitions to Knowledge, Belief and Opinion
2. Provide an outline of Demarcation problem
3. Name and compare the paradigmatic patterns of reasoning
4. Give the definition, enumerate key features of Deductive reasoning and deductive argument (with examples)
5. Give the definition, enumerate key features of Inductive reasoning and deductive argument (with examples)
6. Give the definition, enumerate key features of Abductive reasoning and deductive argument (with examples)
7. Compare the procedures of Validation, Justification and Verification. Clarify the relations between them.
8. Define the key components for the Syntax in Classical Propositional Logic formal language. Build an example.
9. Build a model for Valuations and Interpretations for Classical Propositional Logic.
10. Describe the proof technique Truth Tables and outline the ways they are used in Logic and beyond.
11. Give the definition of Functional Completeness and compare two or more functionally complete sets of logical connectives.
17. Specify the Formal definitions and demonstrate them for CPL.
18. Name and define full list of 16 logical connectives for Propositional Logic and show how they are derived from 5 essentials.
19. Name the differences between formal and informal Proofs.
20. Give a metanalytical review for the types and kinds of Proof Systems.
21. Compare Axiomatic Systems of Proof and Natural Deduction Systems.
22. Describe and give the examples of Natural Deduction Systems and differences between them.
23. Prove the Deduction Theorem in Classical Propositional Logic.
24. Prove the Soundness Theorem in Classical Propositional Logic.
25. Prove the Completeness Theorem in Classical Propositional Logic.
26. Prove the Compactness Theorem in Classical Propositional Logic.
27. Prove the Deduction Theorem in First-Order Predicate Logic.
28. Prove the Soundness Theorem in First-Order Predicate Logic.
29. Prove the Completeness Theorem in First-Order Predicate Logic.
30. Prove the Compactness Theorem in First-Order Predicate Logic.
31. Give an account on Decidability in Classical Propositional Logic.
32. Describe the Decidability Problem for Predicate Logic.
33. Build a Model for Interpretation and Truth-Assignments for Monadic Predicate Logic
34. Build a Model for Interpretation and Truth-Assignments for a full First-Order Predicate Logic
35. Specify the Differences between First-Order Predicate Logic with and without Identity Predicate
36. Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Classical PL system
37. Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for the Monadic Predicate Logic
38. Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for the First-Order Predicate Logic (without Identity)
39. Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for the First-Order Predicate Logic with Identity
40. Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Basic Modal Logic
41. Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Normal Modal Propositional Logic
42. Build an implemented semantic tableaux in a form of 'Truth Tree' for: a) theorem, b) tautology, c) contradiction for any Modal Predicate Logic

43. Enumerate and Specify the functions for all the subsets of Rules in Sequential Calculus in Gerhard Gentzen's deduction system.
44. Name and compare alpha-, beta- and λ - μ F.O. Tableaux Rules subsets of the set of Rules of Decomposition
45. Compare Backward and Forward Reasoning algorithms
46. Define the General Idea behind Many-Valued Logic
47. Describe and Compare FDE-(4-valued) Logic, L3 logic, K3w, K3s and G4/5 logics
48. Represent the Idea behind Possible Worlds Semantics
49. Specify the Differences between First- and Second-Order Predicate Logic
50. Specify the differences for the Initial and Derived Rules of Inference in Natural Deduction systems.

Course Syllabus:

Programmer: lecturer, Candidate of philosophical sciences, Mstislav Kazakov

Approved by Department of Philosophy (protocol № 1 from 31.08.2021)

Agreed with Methodological Council of Igor Sikorsky Kyiv Polytechnic Institute (protocol № 3 from 27.01.2022)